

Probing the SUSY with 10 TeV stop mass in rare decays and CP violation of Kaon

Morimitsu Tanimoto ^a and Kei Yamamoto ^b

^b*Department of Physics, Niigata University,
Niigata 950-2181, Japan*

^b*KEK Theory Center, IPNS, KEK,
Tsukuba, Ibaraki 305-0801, Japan*

Abstract

We probe the SUSY at the 10 TeV scale in the rare decays and the CP violation of the kaon. We focus on the processes of $K_L \rightarrow \pi^0 \nu \bar{\nu}$ and $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ combined with the CP violating parameters ϵ_K and ϵ'_K/ϵ_K . The Z-penguin mediated by the chargino loop cannot enhance $K_L \rightarrow \pi^0 \nu \bar{\nu}$ and $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ because the left-right mixing of the stop is constrained by the 125 GeV Higgs mass. On the other hand, the Z-penguin mediated by the gluino loop can enhance the branching ratios of both $K_L \rightarrow \pi^0 \nu \bar{\nu}$ and $K^+ \rightarrow \pi^+ \nu \bar{\nu}$. The former increases up to more than 1.0×10^{-10} , which is much larger than the SM prediction even if the constraint of ϵ_K is imposed. It is remarkable that the Z-penguin mediated by the gluino loop can enhance simultaneously ϵ'_K/ϵ_K and the branching ratio of $K_L \rightarrow \pi^0 \nu \bar{\nu}$, which increases up to 1.0×10^{-10} . We also study the decay rates of $K_L \rightarrow \mu^+ \mu^-$, $B^0 \rightarrow \mu^+ \mu^-$ and $B_s \rightarrow \mu^+ \mu^-$, which correlate with the $K_L \rightarrow \pi^0 \nu \bar{\nu}$ decay through the Z penguin. It is important to examine the $B^0 \rightarrow \mu^+ \mu^-$ process since we expect the enough sensitivity of this decay mode to the SUSY at LHCb.

1 Introduction

The rare decays and the CP violation of the kaon have given us important constraints for new physics (NP) since the standard model (SM) contributions are suppressed due to the flavor structure of the Cabibbo-Kobayashi-Maskawa (CKM) matrix [1, 2]. Typical examples are the rare decay processes $K_L \rightarrow \pi^0 \nu \bar{\nu}$ and $K^+ \rightarrow \pi^+ \nu \bar{\nu}$, which are clean theoretically [3, 4]. These processes have been considered to be one of the powerful probes of NP [5]-[17]. In order to improve the previous experimental measurements [18, 19], new experiments are going on. One is the J-PARC KOTO experiment, which is to measure the decay rate of $K_L \rightarrow \pi^0 \nu \bar{\nu}$ approaching to the SM predicted precision [20, 21]. Another one is the CERN NA62 experiment to observe the $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ decay [22].

Especially, the $K_L \rightarrow \pi^0 \nu \bar{\nu}$ process is the CP violating one and provides the direct measurement of the CP violating phase in the CKM matrix. On the other hand, the indirect CP violating parameter ϵ_K , which is induced by the $K^0 - \bar{K}^0$ mixing, has given us the precise information of the CP violating phase of the CKM matrix. Another CP violating parameter ϵ'_K/ϵ_K was measured in the $K \rightarrow \pi\pi$ decay. Therefore, the $K_L \rightarrow \pi^0 \nu \bar{\nu}$ process is expected to open the NP window in the CP violation by combining with ϵ_K and ϵ'_K/ϵ_K .

The $K_L \rightarrow \pi^0 \nu \bar{\nu}$ and $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ decays are dominated by the Z-penguin process, which is the flavor changing neutral current (FCNC) through loop diagrams. The Z-penguin process also gives the large contribution to ϵ'_K/ϵ_K due to the enhancement of the $\Delta I = 1/2$ amplitude [23]. Actually, it cancels the dominant QCD penguin contribution significantly in the SM since it has the opposite sign to the QCD penguin amplitude. On the other hand, ϵ_K is given by the box diagram. We expect the deviation from the SM prediction with correlating among $K_L \rightarrow \pi^0 \nu \bar{\nu}$, $K^+ \rightarrow \pi^+ \nu \bar{\nu}$, ϵ_K and ϵ'_K/ϵ_K due to the NP effect. Furthermore, there may be other correlations of NP with the kaon rare decay $K_L \rightarrow \mu^+ \mu^-$ [24] and the B meson rare decays $B^0 \rightarrow \mu^+ \mu^-$, $B_s \rightarrow \mu^+ \mu^-$, which have been observed in the LHCb and CMS experiments [25, 26] since the Z-penguin process also contributes to these processes.

In this work, we discuss the minimal supersymmetric SM (MSSM) as the typical NP. The recent searches for SUSY particles at the LHC give us important constraints. Since the lower bounds of masses of the SUSY particles increase gradually, the gluino mass is supposed to be beyond the scale of 2 TeV [27, 28, 29, 30, 31]. The SUSY models have been also seriously constrained by the Higgs discovery, in which the Higgs mass is 125 GeV [32, 33]. These facts suggest a class of SUSY models with heavy sfermions. If the squark masses are expected to be $\mathcal{O}(10)$ TeV, the lightest Higgs mass can be pushed up to 125 GeV [34], whereas all SUSY particles can be out of the reach of the LHC experiment. Therefore, the indirect search of the SUSY particles becomes important in the low energy flavor physics [35, 36, 37]. We discuss the CP violation related phenomena such as $K_L \rightarrow \pi^0 \nu \bar{\nu}$, $K^+ \rightarrow \pi^+ \nu \bar{\nu}$, ϵ_K and ϵ'_K/ϵ_K in the framework of the high-scale SUSY with $\mathcal{O}(10)$ TeV.

We can also consider the SUSY model with the split-family [38, 39] in which the third family of squarks/sleptons is heavy, $\mathcal{O}(10)$ TeV, while the first and second ones of squarks/sleptons and the gauginos have relatively low masses $\mathcal{O}(1)$ TeV. This model is motivated by the Nambu-Goldstone hypothesis for quarks and leptons in the first two generations [40]. Although there is no signals of the SUSY particles in the LHC experiment at present, this scenario is not conflict with the present bound of the SUSY particles. The split-family model

is consistent with the 125 GeV Higgs mass [32, 33] and the muon $g - 2$ [41]. The stop mass with $\mathcal{O}(10)$ TeV pushes up the lightest Higgs mass to 125 GeV [34]. The deviation from the SM prediction of the muon $g - 2$ [42, 43] is explained by the slepton of the first and second family with the mass less than 1 TeV [39]. Therefore, it is important to examine the split-family model in the rare decays and the CP violation at the low energy [35, 36, 37] as well as the direct search at LHC.

For many years, the rare decays and the CP violation in the K and B mesons have been successfully understood within the framework of the SM, where the source of the CP violation is the Kobayashi-Maskawa (KM) phase [2]. On the other hand, there are new sources of the CP violation if the SM is extended to the SUSY model. For example, the soft squark mass matrices contain the CP violating phases, which contribute to FCNC with the CP violation [44]. Therefore, one expects to discover the SUSY contribution in the CP violating phenomena at the low energy. Actually, we have found that the SUSY contribution could be up to 40% in the observed ϵ_K , but, it is minor in the CP violation of the B meson at the high-scale of 10 – 50 TeV [37]. Moreover, we have also found the sizable contribution of the high-scale SUSY to $K_L \rightarrow \pi^0 \nu \bar{\nu}$ and $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ in the non-minimal flavor violation (non-MFV) scenario [45].

It is also important to take into account of ϵ'_K/ϵ_K because the SM has potential difficulties in describing the data for ϵ'_K/ϵ_K [23]. Therefore, we study ϵ'_K/ϵ_K in the SUSY model with the non-MFV scenario. We discuss $K_L \rightarrow \pi^0 \nu \bar{\nu}$ and $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ with the CP violations, ϵ_K and ϵ'_K/ϵ_K in the framework of the SUSY at $\mathcal{O}(10)$ TeV. In addition, we discuss the SUSY contribution to the decay processes $K_L \rightarrow \mu^+ \mu^-$, $B^0 \rightarrow \mu^+ \mu^-$ and $B_s \rightarrow \mu^+ \mu^-$.

We have already presented the numerical predictions of the branching ratios $K_L \rightarrow \pi^0 \nu \bar{\nu}$ and $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ in ref.[45], where all squarks/sleptons and the gauginos are at $\mathcal{O}(10)$ TeV. However, those numerical results should be revised with the ones of this paper since the relevant constraints are not imposed enough there. In this paper, we also reexamine them comprehensively by taking account of the gluino contribution as well as the chargino one with the large left-right mixing angle of squarks.

Our paper is organized as follows. In section 2, we discuss the formulation of the rare decays, $K_L \rightarrow \pi^0 \nu \bar{\nu}$, $K^+ \rightarrow \pi^+ \nu \bar{\nu}$, $K_L \rightarrow \mu^+ \mu^-$, $B^0 \rightarrow \mu^+ \mu^-$ and $B_s \rightarrow \mu^+ \mu^-$, and CP violations of ϵ_K and ϵ'_K/ϵ_K . Section 3 gives our set-up of the SUSY with the 10 TeV squark masses. In Sec.4, we present our numerical results. Sec.5 is devoted to the summary and discussions. The relevant formulae are presented in Appendices A, B and C.

2 Observables

2.1 $K_L \rightarrow \pi^0 \nu \bar{\nu}$ and $K^+ \rightarrow \pi^+ \nu \bar{\nu}$

Let us begin to discuss the kaon rare decays, $K_L \rightarrow \pi^0 \nu \bar{\nu}$ and $K^+ \rightarrow \pi^+ \nu \bar{\nu}$, which are dominated by the Z-penguin process in the SM. In the estimation of the branching ratios of $K \rightarrow \pi \nu \bar{\nu}$, the hadronic matrix elements can be extracted with the isospin symmetry relation [46, 47]. These processes are theoretically clean because the long-distance contributions are small [14], and then the theoretical uncertainty is estimated below several percent. The accurate measurements of these decay processes provide the crucial tests of the SM. Especially,

the $K_L \rightarrow \pi^0 \nu \bar{\nu}$ process is purely the CP violating one, which can reveal the source of the CP violating phase. The basic formulae are presented in Appendix C1. The SM predictions have been discussed by some works [4, 48, 49]. They are given as ¹:

$$\text{BR}(K_L \rightarrow \pi^0 \nu \bar{\nu})_{\text{SM}} = (3.36 \pm 0.05) \times 10^{-11} \cdot \left[\frac{|V_{ub}|}{3.88 \times 10^{-3}} \right]^2 \left[\frac{|V_{cb}|}{40.7 \times 10^{-3}} \right]^2 \left[\frac{\sin(\gamma)}{\sin(73.2^\circ)} \right]^2, \quad (1)$$

$$\text{BR}(K^+ \rightarrow \pi^+ \nu \bar{\nu})_{\text{SM}} = (8.39 \pm 0.30) \times 10^{-11} \cdot \left[\frac{|V_{cb}|}{40.7 \times 10^{-3}} \right]^{2.8} \left[\frac{\gamma}{73.2^\circ} \right]^{0.74}. \quad (2)$$

On the experimental side, the upper bound of the branching ratio of $K_L \rightarrow \pi^0 \nu \bar{\nu}$ is given by the KEK E391a experiment [18], and the branching ratio of $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ was measured by the BNL E787 and E949 experiments as follows [19]:

$$\text{BR}(K_L \rightarrow \pi^0 \nu \bar{\nu})_{\text{exp}} < 2.6 \times 10^{-8} \text{ (90\%C.L.)}, \quad \text{BR}(K^+ \rightarrow \pi^+ \nu \bar{\nu})_{\text{exp}} = (1.73_{-1.05}^{+1.15}) \times 10^{-10}. \quad (3)$$

At present, the J-PARC KOTO experiment is an in-flight measurement of $K_L \rightarrow \pi^0 \nu \bar{\nu}$ approaching to the SM predicted precision [20, 21], while the CERN NA62 experiment [22] is expected for the precise measurement of the $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ decay.

The SUSY contribution has been studied in many works [50, 51, 52, 53, 54, 55]. The sizable enhancement of these kaon decays was expected through the large left-right mixing of the chargino interaction in $s_L \tilde{t}_i \chi^-$ and $d_L \tilde{t}_i \chi^-$ at the SUSY scale of $\mathcal{O}(1)$ TeV [50, 54]. We find that even at the $\mathcal{O}(10)$ TeV scale, these decays are enhanced through the Z-penguin mediated by the gluino with the large left-right mixing.

2.2 ϵ_K

Let us discuss another CP violating parameter ϵ_K , which was measured precisely. Its hadronic matrix element \hat{B}_K is reliably determined by the lattice calculations as follows [56, 57] :

$$\hat{B}_K = 0.766 \pm 0.010. \quad (4)$$

Another theoretical uncertainty of ϵ_K is also reduced by removing the QCD correction factor of the two charm box diagram [58]. Thus, the accurate estimate of the SM contribution enables us to search for NP such as SUSY. The non-negligible SUSY contribution has been expected in ϵ_K even at the scale of $\mathcal{O}(100)$ TeV [35, 36, 37]. Consequently, ϵ_K gives us one of the most important constraints to predict the SUSY contribution in the $K \rightarrow \pi \nu \bar{\nu}$ decays. In our calculation of ϵ_K , we investigate the SUSY contributions for the box diagram, which is correlated with the $K_L \rightarrow \pi^0 \nu \bar{\nu}$ process directly.

¹In our calculation, we use the CKM elements in the study of the so-called universal unitarity triangle including the data of the CP asymmetry $S_{J/\psi K_S}$ and the mass differences of B mesons without inputting ϵ_K (Strategy S1 in ref.[49]). In this case, the SM prediction for $K \rightarrow \pi \nu \bar{\nu}$ shifts lower.

2.3 ϵ'_K/ϵ_K

The direct CP violation ϵ'_K/ϵ_K is also important to constrain the NP. The basic formula for ϵ'_K/ϵ_K is given as follows [23, 59, 60]:

$$\frac{\epsilon'_K}{\epsilon_K} = \text{Im}(V_{td}V_{ts}^* \cdot F_{\epsilon'}) \quad (5)$$

where

$$F_{\epsilon'} = P_0 + P_X X + P_Y Y + P_Z Z + P_E E, \quad (6)$$

with

$$X = C - 4B^{(u)}, \quad Y = C - B^{(d)}, \quad Z = C + \frac{1}{4}D. \quad (7)$$

Functions B , C , D and E denote the loop-functions including SM and SUSY effects, which come from boxes with external $d\bar{d}(B^{(d)})$, $u\bar{u}(B^{(u)})$, Z-penguin(C), photon-penguin(D) and gluon-penguins(E). The coefficients P_i are given by

$$P_i = r_i^{(0)} + r_i^{(6)} R_6 + r_i^{(8)} R_8, \quad (8)$$

with the non-perturbative parameters $B_6^{(1/2)}$ and $B_8^{(3/2)}$ defined as

$$R_6 \equiv B_6^{(1/2)} \left[\frac{114.54 \text{MeV}}{m_s(m_c) + m_d(m_c)} \right]^2, \quad R_8 \equiv B_8^{(3/2)} \left[\frac{114.54 \text{MeV}}{m_s(m_c) + m_d(m_c)} \right]^2. \quad (9)$$

The numerical values of $r_i^{(0,8,6)}$ are presented in [23].

The most important parameters to predict ϵ'_K/ϵ_K are the non-perturbative parameters $B_6^{(1/2)}$ and $B_8^{(3/2)}$. Recently, the RBC-UKQCD lattice collaboration [61, 62] gives

$$B_6^{(1/2)} = 0.57 \pm 0.15, \quad B_8^{(3/2)} = 0.76 \pm 0.05, \quad (10)$$

which predict $(\epsilon'_K/\epsilon_K)_{\text{SM}} = (1.9 \pm 4.5) \times 10^{-4}$ in the SM [23]. This SM prediction is much smaller than the experimental result [63]

$$(\epsilon'_K/\epsilon_K)_{\text{exp}} = (16.6 \pm 2.3) \times 10^{-4}. \quad (11)$$

This disagreement between the SM prediction and the experimental value may suggest NP in the kaon system, however there are several open questions that have to be answered to conclude it [23]. We use these values of $B_6^{(1/2)}$ and $B_8^{(3/2)}$ with 3σ in our calculation.

The dominant contribution to Z penguin, C , comes from chargino mediated one and gluino mediated one if the large left-right mixing of squarks is allowed. On the other hand, the effect of neutralino are suppressed [50]-[52]. The chargino mediated Z-penguin $C(\chi^\pm)$ and the gluino mediated Z-penguin $C(\tilde{g})$ are given as follows:

$$V_{td}V_{ts}^* C(\chi^\pm) = \frac{1}{8} \left(\frac{4m_W^2}{g_2^2} \right) [P_{\text{ZL}}^{sd}(\chi^\pm)^* + \frac{c_w^2}{s_w^2} P_{\text{ZR}}^{sd}(\chi^\pm)^*], \quad (12)$$

$$V_{td}V_{ts}^* C(\tilde{g}) = \frac{1}{8} \left(\frac{4m_W^2}{g_2^2} \right) [P_{\text{ZL}}^{sd}(\tilde{g})^* + \frac{c_w^2}{s_w^2} P_{\text{ZR}}^{sd}(\tilde{g})^*], \quad (13)$$

where $c_w^2 = \cos^2 \theta_W$ and $s_w^2 = \sin^2 \theta_W$ with the Weinberg angle θ_W , and the Z-penguin amplitudes $P_{\text{ZL(R)}}^{sd}(\chi^\pm)$ and $P_{\text{ZL(R)}}^{sd}(\tilde{g})$ are given in Eqs. (36) and (39) in Appendix B.

The box diagram effect is suppressed compared with the penguin diagram if the SUSY-breaking scale M_S satisfies $M_S \gg m_W$ [53]. Thus, the dominant SUSY contribution to ϵ'_K/ϵ_K is given by the Z-penguin mediated by the chargino and gluino. Therefore, we should consider the correlation between ϵ'_K/ϵ_K and the branching ratio of $K_L \rightarrow \pi^0 \nu \bar{\nu}$.

Let us write ϵ'_K/ϵ_K as

$$\left(\frac{\epsilon'_K}{\epsilon_K}\right) = \left(\frac{\epsilon'_K}{\epsilon_K}\right)_{\text{SM}} + \left(\frac{\epsilon'_K}{\epsilon_K}\right)_Z^L + \left(\frac{\epsilon'_K}{\epsilon_K}\right)_Z^R, \quad (14)$$

where the second and the third terms denote the Z-penguin induced by the left-handed and right-handed interactions of SUSY, respectively. The both contributions are written as follows [24] :

$$\left(\frac{\epsilon'_K}{\epsilon_K}\right)_Z^L + \left(\frac{\epsilon'_K}{\epsilon_K}\right)_Z^R = -2.64 \times 10^3 B_8^{(3/2)} \left[\text{Im} \Delta_L^{sd}(Z) + \frac{c_w^2}{s_w^2} \text{Im} \Delta_R^{sd}(Z) \right], \quad (15)$$

where

$$\Delta_{L(R)}^{sd}(Z) = \frac{g_2}{8\pi^2 c_w} \frac{m_W^2}{2} P_{\text{ZL(R)}}^{sd}. \quad (16)$$

In order to see the correlation between ϵ'_K/ϵ_K and the $K_L \rightarrow \pi^0 \nu \bar{\nu}$ decay, it is helpful to write down the $K_L \rightarrow \pi^0 \nu \bar{\nu}$ amplitude induced by the chargino and gluino mediated Z-penguin in terms of $\Delta_{L(R)}^{sd}(Z)$ as follows:

$$A(K_L \rightarrow \pi^0 \nu \bar{\nu})_Z \sim [\text{Im} \Delta_L^{sd}(Z) + \text{Im} \Delta_R^{sd}(Z)], \quad (17)$$

as seen in Appendix C1.

The Z-penguin amplitude mediated by the chargino dominates the left-handed coupling of the Z boson. Therefore, the chargino contribution to ϵ'_K/ϵ_K is opposite to $K_L \rightarrow \pi^0 \nu \bar{\nu}$. If the Z-penguin mediated by the chargino enhances ϵ'_K/ϵ_K , the $K_L \rightarrow \pi^0 \nu \bar{\nu}$ decay is suppressed considerably. On the other hand, the Z-penguin amplitude mediated by the gluino gives the equal left-handed and right-handed Z couplings. Then, the right-handed Z coupling of the Z-penguin amplitude is by a factor of $c_w^2/s_w^2 \simeq 3.3$ larger than the left-handed one. Therefore, we can obtain the SUSY contribution which can enhance simultaneously ϵ'_K/ϵ_K and the branching ratio for $K_L \rightarrow \pi^0 \nu \bar{\nu}$. Actually, by choosing $\text{Im} \Delta_L^{sd}(Z) > 0$ and $\text{Im} \Delta_R^{sd}(Z) < 0$, the region of

$$|\text{Im} \Delta_R^{sd}(Z)| < \text{Im} \Delta_L^{sd}(Z) < 3.3 |\text{Im} \Delta_R^{sd}(Z)|, \quad (18)$$

can enhance both ϵ'_K/ϵ_K and the branching ratio for $K_L \rightarrow \pi^0 \nu \bar{\nu}$. We discuss this case in our numerical results.

2.4 $K_L \rightarrow \mu^+ \mu^-$, $B^0 \rightarrow \mu^+ \mu^-$ and $B_s \rightarrow \mu^+ \mu^-$ decays

The Z penguin also contributes $K_L \rightarrow \mu^+ \mu^-$, $B^0 \rightarrow \mu^+ \mu^-$ and $B_s \rightarrow \mu^+ \mu^-$ decays. These decay amplitudes are governed by the axial semileptonic operator O_{10} , which is occurred

by the Z-penguin top-loop and the W box diagram in the SM. Those general formulae are presented in Appendix C2. The CMS and LHCb Collaboration have observed the branching ratio for $B_s \rightarrow \mu^+\mu^-$, and $B^0 \rightarrow \mu^+\mu^-$ is also measured [26]:

$$\text{BR}(B_s \rightarrow \mu^+\mu^-)_{\text{exp}} = (2.8_{-0.6}^{+0.7}) \times 10^{-9}, \quad \text{BR}(B^0 \rightarrow \mu^+\mu^-)_{\text{exp}} = (3.9_{-1.4}^{+1.6}) \times 10^{-10}. \quad (19)$$

The SM predictions have been given as [64],

$$\text{BR}(B_s \rightarrow \mu^+\mu^-)_{\text{SM}} = (3.65 \pm 0.23) \times 10^{-9}, \quad \text{BR}(B^0 \rightarrow \mu^+\mu^-)_{\text{SM}} = (1.06 \pm 0.09) \times 10^{-10}. \quad (20)$$

On the other hand, the long-distance effect is expected to be large in the $K_L \rightarrow \mu^+\mu^-$ process [65]. Therefore, it may be difficult to extract the effect of the Z-penguin process. The SM prediction of the short-distance contribution was given as [24],

$$\text{BR}(K_L \rightarrow \mu^+\mu^-)_{\text{SM}} = (0.8 \pm 0.1) \times 10^{-9}. \quad (21)$$

The experimental data of $K_L \rightarrow \mu^+\mu^-$ is [63]

$$\text{BR}(K_L \rightarrow \mu^+\mu^-)_{\text{exp}} = (6.84 \pm 0.11) \times 10^{-9}, \quad (22)$$

from which the constraint on the short-distance contribution has been estimated as [65] :

$$\text{BR}(K_L \rightarrow \mu^+\mu^-)_{\text{SD}} \leq 2.5 \times 10^{-9}. \quad (23)$$

Thus, the SUSY contribution through the Z penguin is expected to be correlated among the rare decays of $K_L \rightarrow \pi^0\nu\bar{\nu}$, $K^+ \rightarrow \pi^+\nu\bar{\nu}$, $K_L \rightarrow \mu^+\mu^-$, $B^0 \rightarrow \mu^+\mu^-$ and $B_s \rightarrow \mu^+\mu^-$ as well as the CP violations of ϵ_K and ϵ'_K/ϵ_K .

3 SUSY flavor mixing

Recent LHC results for the SUSY search may suggest the high-scale SUSY, $\mathcal{O}(10 - 1000)$ TeV [35, 36, 37] since the lower bounds of the gluino mass and squark masses are close to 2 TeV. Taking account of these recent results, we consider the possibility of the high-scale SUSY at 10 TeV, in which the $K \rightarrow \pi\nu\bar{\nu}$ decays and ϵ'_K/ϵ_K with the constraint of ϵ_K are discussed.

We also consider the split-family model, which has the specific spectrum of the SUSY particles [38, 39]. This model is motivated by the Nambu-Goldstone hypothesis for quarks and leptons in the first two generations [40]. Therefore, the third family of squark/slepton is heavy, for example, $\mathcal{O}(10)$ TeV while the first and second family squarks/slepton have relatively low masses $\mathcal{O}(1)$ TeV. The masses of bino and wino are assumed to be small close to the experimental lower bound, less than 1 TeV. The model was at first discussed in the $B_s - \bar{B}_s$ mixing [38]. It explained successfully both the 125 GeV Higgs mass and the muon $g - 2$ simultaneously [39]. The stop mass with $\mathcal{O}(10)$ TeV pushes up the Higgs mass to 125 GeV. The deviation of the muon $g - 2$ is explained by the slepton of the first and second families with the mass less than 1 TeV. Since the squark masses of the first and second families

are also relatively low as well as the sleptons, we expect the SUSY contribution in the kaon system becomes large.

The new flavor mixing and CP violation effect are induced through the quark-squark-gaugino and the lepton-slepton-gaugino couplings. The 6×6 squark mass matrix M_q^2 in the super-CKM basis is diagonalized to the mass eigenstate basis in terms of the rotation matrix $\Gamma^{(q)}$ as

$$m_q^2 = \Gamma^{(q)} M_q^2 \Gamma^{(q)\dagger}, \quad (24)$$

where $\Gamma^{(q)}$ is 6×6 unitary matrix, and it is decomposed into 3×6 matrices as $\Gamma^{(q)} = (\Gamma_L^{(q)}, \Gamma_R^{(q)})$. The explicit matrix is shown in Appendix A. We introduce twelve mixing parameters $s_{12}^{qL,qR}$, $s_{23}^{qL,qR}$ and $s_{13}^{qL,qR}$, where $q = u, d$ for the squark mixing. In addition, we also introduce left-right (LR) mixing angles $\theta_{LR}^{t,b}$.

In practice, we take $s_{12}^{qL,qR} = 0$, which is motivated by the almost degenerate squark masses of the first and the second families to protect the large contribution to the $K^0 - \bar{K}^0$ mass difference ΔM_K . It is also known that the single mixing effect of $s_{12}^{qL,qR}$ to $K \rightarrow \pi \nu \bar{\nu}$ is minor [50]. Actually, we have checked numerically that the contribution of $s_{12}^{qL,qR} = 0 \sim 0.3$ is negligibly small. There also appear the phases ϕ_{ij}^{qL} and ϕ_{ij}^{qR} associated with the mixing angles, which bring new sources of the CP violations. In our work, we treat those mixing parameters and phases as free parameters in the framework of the non-MFV scenario.

Since the Z-penguin processes give dominant contribution for $K \rightarrow \pi \nu \bar{\nu}$ and ϵ'_K/ϵ_K , we calculate the Z-penguin mediated by the chargino and gluino. The interaction is presented in Appendix B. The relevant parameters are presented in the following section.

4 Numerical analysis

4.1 Set-up of parameters

Let us discuss the decay rates of $K_L \rightarrow \pi^0 \nu \bar{\nu}$ and $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ processes by choosing a sample of the mass spectrum in the high-scale SUSY model at $\mathcal{O}(10)$ TeV. The enhancements of these kaon rare decays require the large left-right mixing with the large squark flavor mixing. In order to show our results clearly, we take a simple set-up for the high-scale SUSY model as follows:

- We fix the gluino, wino and bino masses $M_i (i = 3, 2, 1)$ with μ and $\tan\beta$ as:

$$M_3 = 10 \text{ TeV}, \quad M_2 = 3.3 \text{ TeV}, \quad M_1 = 1.6 \text{ TeV}, \quad \mu = 10 \text{ TeV}, \quad \tan\beta = 3, \quad (25)$$

for the high-scale SUSY.

- We take the masses of stop \tilde{t}_1, \tilde{t}_2 , and sbottom \tilde{b}_1, \tilde{b}_2 as a sample set

$$m_{\tilde{t}_1} = 10 \text{ TeV}, \quad m_{\tilde{t}_2} = 15 \text{ TeV}, \quad m_{\tilde{b}_1} = 10 \text{ TeV}, \quad m_{\tilde{b}_2} = 15 \text{ TeV}. \quad (26)$$

On the other hand, we take the masses of the first and second family up-type and down-type squarks around 15 TeV within 5 – 15% relevantly. This mass spectrum of the first and second families does not so change our numerical results because the third

family squarks dominate the Z-penguin induced by the chargino and gluino interactions in our model.

- We take the left-right mixing angles

$$\theta_{LR}^t = 0.07 \quad \text{and} \quad \theta_{LR}^b = 0.1 - 0.3, \quad (27)$$

where θ_{LR}^t is estimated by input of the stop masses in Eq.(26) with the large A term, which is constrained by the 125 GeV Higgs mass due to the large radiative correction [34]. On the other hand, there is no strong constraint for the left-right mixing of the down squarks from the B meson experiments in the region of $\mathcal{O}(10)$ TeV². Therefore, we take rather large values to see the enhancement of the $K_L \rightarrow \pi^0 \nu \bar{\nu}$ decay.

- The flavor mixing parameters s_{ij}^{qL} and s_{ij}^{qR} of the up and down sectors are free parameters, and are varied in

$$s_{i3}^{uL}, s_{i3}^{dL} = 0 \sim 0.3 \quad (i = 1, 2), \quad s_{i3}^{uR}, s_{i3}^{dR} = 0 \sim 0.3 \quad (i = 1, 2), \quad (28)$$

where the upper bound 0.3 is given by the experimental constraint of the $K^0 - \bar{K}^0$ mass difference ΔM_K . As discussed in the previous section, we ignore the mixing between the first and second family of squarks, s_{12}^{qL} , and then, can avoid the large contribution from s_{12}^{qL} to ΔM_K . This single mixing effect of s_{12}^{qL} to the Z-penguin mediated by the chargino is known to be minor compared with double mixing effect [50, 54]. Namely, the SUSY contributions of the $K_L \rightarrow \pi^0 \nu \bar{\nu}$ and $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ processes are dominated by the double mixing of the stop and sbottom.

- The phase parameters $\phi_{13}^{qL(R)}$ and $\phi_{23}^{qL(R)}$ are also free parameters. We scan them in $-\pi \sim \pi$ randomly.
- We neglect the minor contribution from the slepton and sneutrino. We also neglect the charged Higgs contribution, which is tiny due to the CKM mixing.
- For non-perturbative parameters $B_6^{(1/2)}$ and $B_8^{(3/2)}$, which are key ones to estimate ϵ'_K/ϵ_K , we use the RBC-UKQCD result $B_6^{(1/2)} = 0.57 \pm 0.15$ and $B_8^{(3/2)} = 0.76 \pm 0.05$ in Eq.(10). We scan them within the 3σ error-bar.

We use the CKM elements $|V_{cb}|$, $|V_{ub}|$, $|V_{td}|$ in ref.[49] with 3σ error bars, which are obtained in the framework of the SM. If there is a large SUSY contribution to the kaon and the B meson systems, the values of the CKM elements may be changed. Actually, the SUSY contribution is comparable to the SM one for ϵ_K in our following numerical analyses, however, very small for the CP violations and the mass differences of the B mesons at the $\mathcal{O}(10)$ TeV scale of squarks [37]. We use the CKM element in the study of the unitarity triangle including the data of the CP asymmetries and the mass differences of B mesons without inputting ϵ_K (Strategy S1 in ref.[49]).

²The metastability of vacuum can also constrain the left-right mixing for the down squark sector [66]. In order to justify our set-up of the left-right mixing angle, the more precise analysis of the vacuum stability is important.

4.2 Results in the SUSY at 10 TeV

Let us discuss the case of the high-scale SUSY, where all squark/slepton are at the 10 TeV scale.

At first, we discuss the contribution of the Z-penguin induced by the chargino to the $K_L \rightarrow \pi^0 \nu \bar{\nu}$ and $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ processes. In this case, the left-right mixing of the up squark sector controls the magnitude of the Z-penguin amplitude. Since the A term is considerably constrained by the 125 GeV Higgs mass, the left-right mixing angle cannot be large in our mass spectrum, at most $\theta_{LR}^t = 0.07$ as presented in the above set-up. Therefore, we cannot obtain the enhancement of those processes³. Actually, the predicted branching ratios of $K_L \rightarrow \pi^0 \nu \bar{\nu}$ and $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ deviate from the prediction of the SM with of order 10%. Thus, we conclude that the Z-penguin mediated by chargino cannot bring large enhancement for the $K_L \rightarrow \pi^0 \nu \bar{\nu}$ and $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ decays due to the constraint of the 125 GeV Higgs mass. This result is consistent with the recent work [67], where the metastability of vacuum constrains the left-right mixing for the up squark sector.

On the other hand, the Z-penguin induced by the gluino could be large due to the large down-type left-right mixing $\theta_{LR}^b = 0.1 - 0.3$. In our set-up of parameters, we show the predicted branching ratios, $\text{BR}(K_L \rightarrow \pi^0 \nu \bar{\nu})$ versus $\text{BR}(K^+ \rightarrow \pi^+ \nu \bar{\nu})$ in fig.1, where the mixing $s_{13}^{dL,dR}$ and $s_{23}^{dL,dR}$ are scanned in $0 - 0.3$ and the left-right mixing angle θ_{LR}^b is fixed to be 0.3. Here the Grossman-Nir bound is shown by the slant green line [68]. In order to see the θ_{LR}^b dependence, we also present the $\text{BR}(K_L \rightarrow \pi^0 \nu \bar{\nu})$ versus $\text{BR}(K^+ \rightarrow \pi^+ \nu \bar{\nu})$ in fig.2 and fig.3, in which $\theta_{LR}^b = 0.2$ and 0.1 are fixed respectively. As seen in figs.1-3, the branching ratio of $\text{BR}(K_L \rightarrow \pi^0 \nu \bar{\nu})$ depends on considerably the left-right mixing angle θ_{LR}^b . The enhancement of $\text{BR}(K_L \rightarrow \pi^0 \nu \bar{\nu})$ requires the left-right mixing angle to be larger than 0.1.

Though the constraint of the experimental value of ϵ_K is important, it is not imposed in figs.1-3. Let us take account of ϵ_K . The gluino contribution to ϵ_K depends on the phase differences of $\phi_{13}^{dL(dR)}$ and $\phi_{23}^{dL(dR)}$, which are associated with flavor mixing angles. In order to avoid the large contribution of the relatively light squarks to ϵ_K , the phases $\phi_{13}^{dL,dR} - \phi_{23}^{dL,dR}$ should be tuned near $n \times \pi/2$ ($n = -2, -1, 0, 1, 2$)⁴. For the phase cycle in the branching ratio, $\text{BR}(K_L \rightarrow \pi^0 \nu \bar{\nu})$ is a half of the one in ϵ_K . Therefore, the enhancement of $\text{BR}(K_L \rightarrow \pi^0 \nu \bar{\nu})$ is realized at $\phi_{13}^{dL} - \phi_{23}^{dL} \simeq \pi/2$ and $\phi_{13}^{dR} - \phi_{23}^{dR} \simeq -\pi/2$, where ϵ_K is enough suppressed. At $\phi_{13}^{dL} - \phi_{23}^{dL} \simeq -\pi/2$ and $\phi_{13}^{dR} - \phi_{23}^{dR} \simeq \pi/2$, the SUSY contribution to the $K_L \rightarrow \pi^0 \nu \bar{\nu}$ process is the opposite to the SM one, and then the branching ratio is suppressed compared with the SM prediction.

We show the predicted region for $\text{BR}(K_L \rightarrow \pi^0 \nu \bar{\nu})$ versus $\text{BR}(K^+ \rightarrow \pi^+ \nu \bar{\nu})$, with imposing ϵ_K where $\theta_{LR}^b = 0.3$ is fixed in fig.4. There are two direction in the predicted plane of $\text{BR}(K_L \rightarrow \pi^0 \nu \bar{\nu})$ versus $\text{BR}(K^+ \rightarrow \pi^+ \nu \bar{\nu})$. The direction of the enhancement of $\text{BR}(K_L \rightarrow \pi^0 \nu \bar{\nu})$ corresponds to $\phi_{13}^{dL} - \phi_{23}^{dL} \simeq -\pi/2$ and $\phi_{13}^{dR} - \phi_{23}^{dR} \simeq \pi/2$, and the enhancement of $\text{BR}(K^+ \rightarrow \pi^+ \nu \bar{\nu})$ to $\phi_{13}^{dL,dR} - \phi_{23}^{dL,dR} \simeq 0, \pi$.

³If we take the smaller mass for $m_{\tilde{t}_2}$ in Eq.(26), for example, 12 TeV, the left-right mixing angle can be chosen to be larger than 0.1. However, the contribution of $m_{\tilde{t}_1}$ is canceled by the one of $m_{\tilde{t}_2}$ due to the small mass difference significantly.

⁴The interpretation of the relation between the phase dependence of $K \rightarrow \pi \nu \bar{\nu}$ and the one of ϵ_K was discussed in ref. [15].

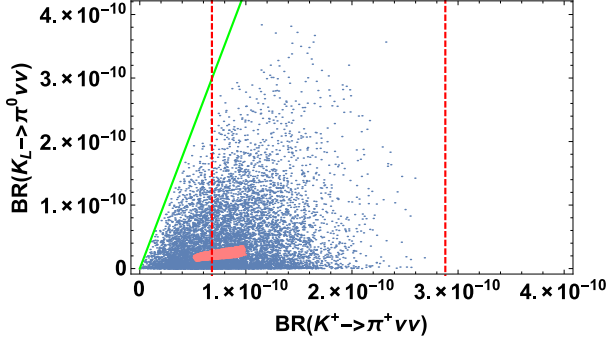


Figure 1: The predicted region for $\text{BR}(K_L \rightarrow \pi^0 \nu \bar{\nu})$ versus $\text{BR}(K^+ \rightarrow \pi^+ \nu \bar{\nu})$ without imposing ϵ_K where $\theta_{LR}^b = 0.3$. The green line corresponds to the Grossman-Nir bound. The dashed red lines denote the 1σ experimental bounds for $\text{BR}(K^+ \rightarrow \pi^+ \nu \bar{\nu})$. The pink denotes the SM prediction.

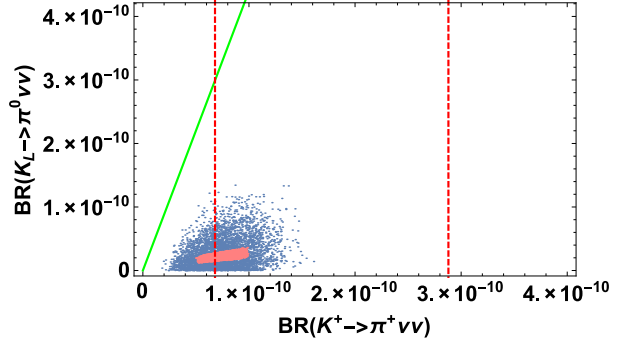


Figure 2: The predicted region for $\text{BR}(K_L \rightarrow \pi^0 \nu \bar{\nu})$ versus $\text{BR}(K^+ \rightarrow \pi^+ \nu \bar{\nu})$, without imposing ϵ_K , where $\theta_{LR}^b = 0.2$. Notations are same as in Figure 1.

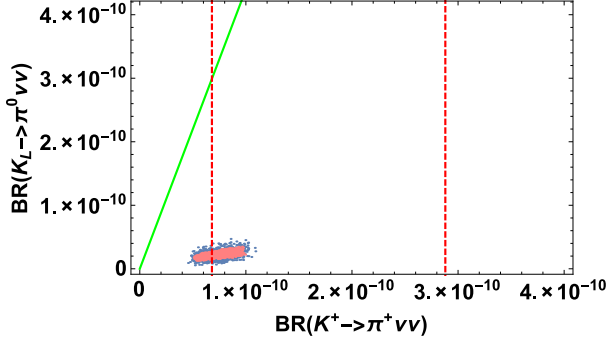


Figure 3: The predicted region for $\text{BR}(K_L \rightarrow \pi^0 \nu \bar{\nu})$ versus $\text{BR}(K^+ \rightarrow \pi^+ \nu \bar{\nu})$, without imposing ϵ_K , where $\theta_{LR}^b = 0.1$. Notations are same as in Figure 1.

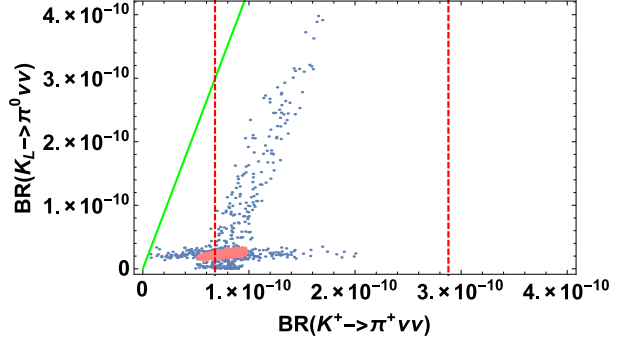


Figure 4: The predicted region for $\text{BR}(K_L \rightarrow \pi^0 \nu \bar{\nu})$ versus $\text{BR}(K^+ \rightarrow \pi^+ \nu \bar{\nu})$, with imposing ϵ_K , where $\theta_{LR}^b = 0.3$. Notations are same as in Figure 1.

As a result, it is found that $\text{BR}(K_L \rightarrow \pi^0 \nu \bar{\nu})$ can be enhanced up to 4×10^{-10} , which is much larger than the SM one, with satisfying the ϵ_K constraint.

We comment on the constraint from $K^0 - \bar{K}^0$ mass difference ΔM_K . Our SUSY contribution of $\Delta M_K(\text{SUSY})$ is comparable with the SM contribution $\Delta M_K(\text{SM})$. It is possible to fit the following condition keeping the enhancement of $\text{BR}(K_L \rightarrow \pi^0 \nu \bar{\nu})$:

$$\frac{\Delta M_K}{\Delta M_K(\text{SM})} = 0.75 \sim 1.25, \quad (29)$$

which is the criterion of the allowed NP contribution in ref. [69]. We also estimate the SUSY contribution to ΔM_{B^0} and ΔM_{B_s} , which are at most 10% of the SM.

Let us discuss the correlation between $\text{BR}(K_L \rightarrow \pi^0 \nu \bar{\nu})$ and ϵ'_K/ϵ_K . As discussed in subsection 2.3, both processes come from the imaginary part of the same Z-penguin, and

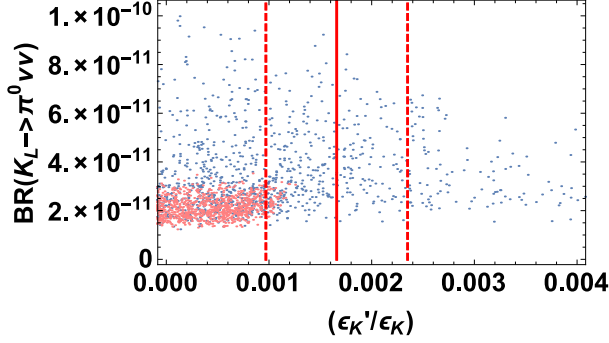


Figure 5: The predicted $\text{BR}(K_L \rightarrow \pi^0 \nu \bar{\nu})$ versus ϵ'_K/ϵ_K , where the Zsd coupling satisfies the condition of eq.(18). The vertical solid red line denotes the central value of the experimental data, and the dashed ones denote the experimental bounds with 3σ for ϵ'_K/ϵ_K . The pink denotes the SM prediction.

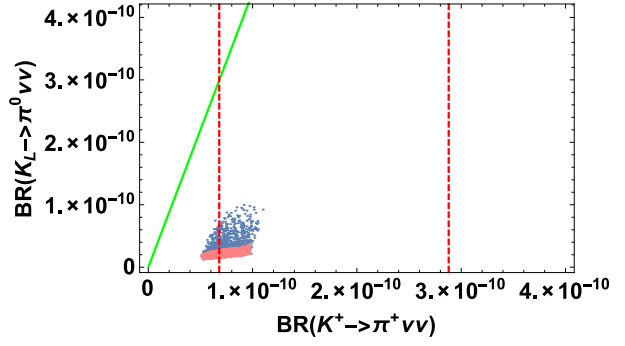


Figure 6: The predicted region for $\text{BR}(K_L \rightarrow \pi^0 \nu \bar{\nu})$ versus $\text{BR}(K^+ \rightarrow \pi^+ \nu \bar{\nu})$, where the Zsd coupling satisfies the condition of eq.(18). Notations are same as in Figure 1.

can be enhanced simultaneously once the condition Eq.(18) is imposed. In fig.5, we show the correlation between $\text{BR}(K_L \rightarrow \pi^0 \nu \bar{\nu})$ and ϵ'_K/ϵ_K , where Zsd coupling satisfies the condition of eq.(18). The constraint from ϵ_K is also imposed. It is remarkable that the Z-penguin mediated by the gluino enhances simultaneously ϵ'_K/ϵ_K and the branching ratio for $K_L \rightarrow \pi^0 \nu \bar{\nu}$. While the estimated ϵ'_K/ϵ_K fits the observed value, the branching ratio of $K_L \rightarrow \pi^0 \nu \bar{\nu}$ increases up to 1.0×10^{-10} . In this region, the phase of $\text{Im}\Delta_L^{sd}$ and $\text{Im}\Delta_R^{sd}$ becomes opposite, so the enhanced region of $\text{BR}(K_L \rightarrow \pi^0 \nu \bar{\nu})$ is somewhat reduced by the cancellation between the left-handed coupling of Z and the right-handed one partially, compared with the result in fig.4.

The real part of Δ_L^{sd} and Δ_R^{sd} are small sufficiently since $\phi_{13}^{dL,dR} - \phi_{23}^{dL,dR} \simeq \pm\pi/2$ are taken. Therefore, the SUSY contribution does not spoil the agreement of the real part of the $K \rightarrow \pi\pi$ amplitude in the SM with the experimental data.

In fig.6, we show the correlation between $\text{BR}(K_L \rightarrow \pi^0 \nu \bar{\nu})$ and $\text{BR}(K^+ \rightarrow \pi^+ \nu \bar{\nu})$. In the parameter region where $\text{BR}(K_L \rightarrow \pi^0 \nu \bar{\nu})$ and ϵ'_K/ϵ_K are enhanced, the branching ratio of $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ is not deviated from the SM. It is understandable because $\phi_{13}^{dL,dR} - \phi_{23}^{dL,dR} \simeq \pm\pi/2$ is taken in order to enhance $\text{BR}(K_L \rightarrow \pi^0 \nu \bar{\nu})$ with the ϵ_K constraint. On the other hand, $\text{BR}(K^+ \rightarrow \pi^+ \nu \bar{\nu})$ is dominated by the considerably sizable real part of the SM. The addition of the imaginary part of the SUSY contribution does not change the SM prediction significantly.

The Z-penguin process also contributes to another kaon rare decay $K_L \rightarrow \mu^+ \mu^-$, and the B meson rare decays, $B^0 \rightarrow \mu^+ \mu^-$ and $B_s \rightarrow \mu^+ \mu^-$. Therefore, we expect them to correlate with the $K \rightarrow \pi \nu \bar{\nu}$ decays. In the $K_L \rightarrow \mu^+ \mu^-$ process, the long-distance effect is estimated to be large in ref. [65]. Therefore, we only discuss the short-distance effect, which is dominated by the Z-penguin. We show $\text{BR}(K_L \rightarrow \pi^0 \nu \bar{\nu})$ versus $\text{BR}(K_L \rightarrow \mu^+ \mu^-)$ in fig.7, where the constraint from ϵ_K is imposed. It is noticed that the predicted value almost satisfies the bound for the short-distance contribution in Eq.(23), presented as the red line.

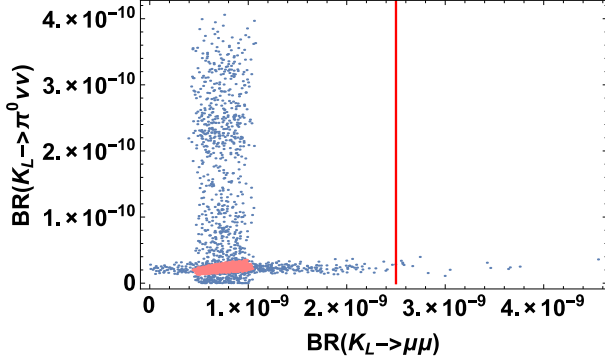


Figure 7: The predicted $\text{BR}(K_L \rightarrow \pi^0 \nu \bar{\nu})$ versus $\text{BR}(K_L \rightarrow \mu^+ \mu^-)$. The pink denotes the SM with 3σ . The solid red line denotes the bound for the short-distance contribution.

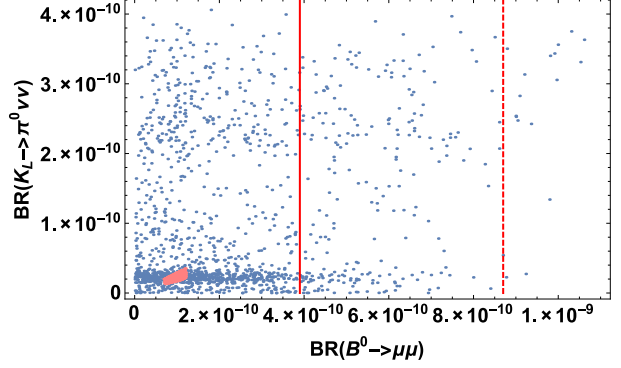


Figure 8: The predicted $\text{BR}(K_L \rightarrow \pi^0 \nu \bar{\nu})$ versus $\text{BR}(B^0 \rightarrow \mu^+ \mu^-)$. The solid red line denotes the central value of the experimental data, and the dashed one denotes the experimental upper bound with 3σ . The pink denotes the SM with 3σ .

The clear correlation between two branching ratios is understandable because $\text{BR}(K_L \rightarrow \mu^+ \mu^-)$ is sensitive only to the real part of Z-couplings. When the enhancement of $\text{BR}(K_L \rightarrow \pi^0 \nu \bar{\nu})$ is found in the future, $\text{BR}(K_L \rightarrow \mu^+ \mu^-)$ remains to be less than 10^{-9} . On the other hand, $\text{BR}(K_L \rightarrow \mu^+ \mu^-)$ is larger than 10^{-9} , there is no enhancement of $\text{BR}(K_L \rightarrow \pi^0 \nu \bar{\nu})$. This relation is testable in the future experiments.

We also show $\text{BR}(K_L \rightarrow \pi^0 \nu \bar{\nu})$ versus $\text{BR}(B^0 \rightarrow \mu^+ \mu^-)$ in fig.8. We can expect the enhancement of $\text{BR}(B^0 \rightarrow \mu^+ \mu^-)$ in our set-up even if $\text{BR}(K_L \rightarrow \pi^0 \nu \bar{\nu})$ is comparable to the SM one. Since LHCb will observe the $\text{BR}(B^0 \rightarrow \mu^+ \mu^-)$ [70], this result is the attractive one in our model.

On the other hand, we do not see the correlation between $\text{BR}(K_L \rightarrow \pi^0 \nu \bar{\nu})$ and $\text{BR}(B_s \rightarrow \mu^+ \mu^-)$ since the SM component of $\text{BR}(B_s \rightarrow \mu^+ \mu^-)$ is relatively large compared with $B^0 \rightarrow \mu^+ \mu^-$. The enhancement of the $K_L \rightarrow \pi^0 \nu \bar{\nu}$ decay rate is still consistent with the present experimental data of $\text{BR}(B_s \rightarrow \mu^+ \mu^-)$.

4.3 Results in the split-family with 10 TeV stop and sbottom

Let us discuss the case of the the split-family of SUSY with 10 TeV stop and sbottom, where 1st and 2nd family squark masses are around 2 TeV. The constraint of ϵ_K is seriously tight for the CP violating phases associated with the squark mixing in the split-family SUSY model. Moreover, the $|\Delta F| = 2$ processes receive too large contributions from the the first and second squarks because they are relatively light, at $\mathcal{O}(1)$ TeV. Actually, ΔM_K , ΔM_{B^0} and ΔM_{B_s} are predicted as

$$\frac{\Delta M_K}{\Delta M_K(SM)} \simeq 400, \quad \frac{\Delta M_{B^0}}{\Delta M_{B^0}(SM)} \simeq 50, \quad \frac{\Delta M_{B_s}}{\Delta M_{B_s}(SM)} \simeq 3. \quad (30)$$

In addition, the large left-right mixing generates large contributions to the $b \rightarrow s \gamma$ decay, therefore, the left-right mixing angle is severely constrained by experimental data of $b \rightarrow s \gamma$.

Therefore, it is impossible to realize the enhancement of $\text{BR}(K_L \rightarrow \pi^0 \nu \bar{\nu})$ in the split-family model satisfying constraints of $|\Delta F| = 1, 2$ transitions in the kaon and the B meson systems.

4.4 EDMs of neutron and mercury

Finally, we add a comment on the electric dipole moments (EDMs) of the neutron and the mercury (Hg), d_n and d_{Hg} , which arise through the chromo-EDM of the quarks, d_q^C due to the gluino-squark mixing [71]-[76]. If both left-handed and right-handed mixing angles are taken to be large such as $s_{13}^{dL} = s_{13}^{dR} \simeq 0.3$ or $s_{23}^{dL} = s_{23}^{dR} \simeq 0.3$ with the large left-right mixing, d_n and d_{Hg} are predicted to be one and two orders larger than the experimental upper bound, respectively [63], $|d_n| < 0.29 \times 10^{-25} \text{e} \cdot \text{cm}$ and $|d_{Hg}| < 3.1 \times 10^{-29} \text{e} \cdot \text{cm}$.

However, there still remains the freedom of phase parameters. For example, by tuning ϕ_{i3}^{dL} and ϕ_{i3}^{dR} ($i = 1, 2$) under the constraint from ϵ_K , we can suppress the EDMs enough. This tuning do not spoil our numerical results above.

5 Summary and discussions

In order to probe the SUSY at the 10 TeV scale, we have studied the processes of $K_L \rightarrow \pi^0 \nu \bar{\nu}$ and $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ combined with the CP violating parameters ϵ_K and ϵ'_K/ϵ_K . The Z-penguin mediated by the chargino loop cannot enhance $K_L \rightarrow \pi^0 \nu \bar{\nu}$ and $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ because the left-right mixing of the stop is constrained by the 125 GeV Higgs mass. On the other hand, the Z-penguin mediated by the gluino loop can enhance the branching ratios of both $K_L \rightarrow \pi^0 \nu \bar{\nu}$ and $K^+ \rightarrow \pi^+ \nu \bar{\nu}$, where the former increases more than 1.0×10^{-10} , much larger than the SM prediction even if the constraint of ϵ_K is imposed. Thus, the $K_L \rightarrow \pi^0 \nu \bar{\nu}$ and $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ decays provide us very important information to probe the SUSY.

It is remarkable that the Z-penguin mediated by the gluino loop can enhance simultaneously ϵ'_K/ϵ_K and the branching ratio for $K_L \rightarrow \pi^0 \nu \bar{\nu}$. While the estimated ϵ'_K/ϵ_K fits the observed value, the branching ratio of $K_L \rightarrow \pi^0 \nu \bar{\nu}$ increses up to 1.0×10^{-10} .

We have also studied the decay rates of $K_L \rightarrow \mu^+ \mu^-$, $B^0 \rightarrow \mu^+ \mu^-$ and $B_s \rightarrow \mu^+ \mu^-$, which correlate with the $K_L \rightarrow \pi^0 \nu \bar{\nu}$ decay through the Z-penguin. Especially, it is important to examine the $B^0 \rightarrow \mu^+ \mu^-$ decay carefully since we can expect the enough sensitivity of the SUSY in this decay mode at LHCb .

We have also discussed them in the split-family model of SUSY, where the third family of squarks/sleptons is heavy, $\mathcal{O}(10)$ TeV, while the first and second ones of squarks/sleptons and the gauginos have relatively low masses of $\mathcal{O}(1)$ TeV. The constraint of ϵ_K is much seriously tight for the CP violating phases associated with the squark mixing in the split-family SUSY model. Moreover, the $|\Delta F| = 2$ processes receive too large contributions from the first and second family squarks because they are relatively light, at $\mathcal{O}(1)$ TeV. Therefore, it is impossible to realize the enhancement of $\text{BR}(K_L \rightarrow \pi^0 \nu \bar{\nu})$ in the split-family model.

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Appendix A : Squark flavor mixing matrix

The flavor mixing and CP violation are induced through the quark-squark-gaugino and the lepton-slepton-gaugino couplings. The Lagrangian of the gaugino-quark-squark interaction is written as

$$\mathcal{L}_{\text{int}}(\tilde{G}q\tilde{q}) = -i\sqrt{2}g_{1,2,3} \sum_{\{q\}} \tilde{q}_i^*(T^a) \tilde{G}^a \left[(\Gamma_L^{(q)})_{ij} \mathbf{L} + (\Gamma_R^{(q)})_{ij} \mathbf{R} \right] q_j + \text{H.c.}, \quad (31)$$

where \tilde{G}^a is the gaugino field, T^a is the generator of the gauge group, and \mathbf{L}, \mathbf{R} are projection operators. The left-handed and right-handed mixing matrixes $\Gamma_L^{(q)}$ and $\Gamma_R^{(q)}$ diagonalizes the 6×6 squark mass matrix M_q^2 in the super-CKM basis to the mass eigenstate basis as follows:

$$M_q^2 = \Gamma^{(q)\dagger} \text{diag}(m_{\tilde{q}}^2) \Gamma^{(q)} = \begin{pmatrix} M_{LL}^2 & M_{LR}^2 \\ M_{RL}^2 & M_{RR}^2 \end{pmatrix}, \quad (32)$$

where $\Gamma^{(q)}$ is the 6×6 unitary matrix, and it is decomposed into the 3×6 matrices as $\Gamma^{(q)} = (\Gamma_L^{(q)}, \Gamma_R^{(q)})$. The squark mass matrix M_q^2 in the super-CKM basis is the same as that in the SLHA notation [77]. We write $\Gamma_{L,R}^{(q)}$ as follows:

$$\Gamma_L^{(q)} = \begin{pmatrix} c_{13}^{qL} & 0 & s_{13}^{qL} e^{-i\phi_{13}^{qL}} c_{\theta_{LR}^q} & 0 & 0 & -s_{13}^{qL} e^{-i\phi_{13}^{qL}} s_{\theta_{LR}^q} e^{i\phi_{LR}^q} \\ -s_{23}^{qL} s_{13}^{qL} e^{i(\phi_{13}^{qL} - \phi_{23}^{qL})} & c_{23}^{qL} & s_{23}^{qL} c_{13}^{qL} e^{-i\phi_{23}^{qL}} c_{\theta_{LR}^q} & 0 & 0 & -s_{23}^{qL} c_{13}^{qL} e^{-i\phi_{23}^{qL}} s_{\theta_{LR}^q} e^{i\phi_{LR}^q} \\ -s_{13}^{qL} c_{23}^{qL} e^{i\phi_{13}^{qL}} & -s_{23}^{qL} e^{i\phi_{23}^{qL}} & c_{13}^{qL} c_{23}^{qL} c_{\theta_{LR}^q} & 0 & 0 & -c_{13}^{qL} c_{23}^{qL} s_{\theta_{LR}^q} e^{i\phi_{LR}^q} \end{pmatrix}^T, \quad (33)$$

$$\Gamma_R^{(q)} = \begin{pmatrix} 0 & 0 & s_{13}^{qR} s_{\theta_{LR}^q} e^{-i\phi_{13}^{qR}} e^{-i\phi_{LR}^q} & c_{13}^{qR} & 0 & s_{13}^{qR} e^{-i\phi_{13}^{qR}} c_{\theta_{LR}^q} \\ 0 & 0 & s_{23}^{qR} c_{13}^{qR} s_{\theta_{LR}^q} e^{-i\phi_{23}^{qR}} e^{-i\phi_{LR}^q} & -s_{13}^{qR} s_{23}^{qR} e^{i(\phi_{13}^{qR} - \phi_{23}^{qR})} & c_{23}^{qR} & s_{23}^{qR} c_{13}^{qR} e^{-i\phi_{23}^{qR}} c_{\theta_{LR}^q} \\ 0 & 0 & c_{13}^{qR} c_{23}^{qR} s_{\theta_{LR}^q} e^{-i\phi_{13}^{qR}} & -s_{13}^{qR} c_{23}^{qR} e^{i\phi_{13}^{qR}} & -s_{23}^{qR} e^{i\phi_{23}^{qR}} & c_{13}^{qR} c_{23}^{qR} c_{\theta_{LR}^q} \end{pmatrix}^T, \quad (33)$$

where we use abbreviations $c_{ij}^{qL,qR} = \cos \theta_{ij}^{qL,qR}$, $s_{ij}^{qL,qR} = \sin \theta_{ij}^{qL,qR}$, $c_{\theta^q} = \cos \theta^q$ and $s_{\theta^q} = \sin \theta^q$ with $\theta_{ij}^{qL,qR}$ being the mixing angles between i -th and j -th families of squarks. In these mixing matrices, we take $s_{12}^{qL,qR} = 0$.

The 3×3 submatrix M_{LR}^2 is given as follows:

$$M_{LR}^2 = (m_{\tilde{q}_{3,1}}^2 - m_{\tilde{q}_{3,2}}^2) \cos \theta_{LR}^{q3} \sin \theta_{LR}^{q3} e^{i\phi_{LR}^q} \times \begin{pmatrix} s_{13}^{qL} s_{13}^{qR} e^{i(\phi_{13}^{qL} - \phi_{13}^{qR})} & c_{13}^{qR} s_{13}^{qL} s_{13}^{qR} e^{i(\phi_{13}^{qL} - \phi_{13}^{qR})} & c_{13}^{qR} c_{23}^{qR} s_{13}^{qL} e^{i\phi_{13}^{qL}} \\ c_{13}^{qL} s_{13}^{qR} s_{23}^{qL} e^{i(\phi_{23}^{qL} - \phi_{13}^{qR})} & c_{13}^{qL} c_{13}^{qR} s_{23}^{qL} s_{23}^{qR} e^{i(\phi_{23}^{qL} - \phi_{23}^{qR})} & c_{13}^{qL} c_{13}^{qR} s_{23}^{qL} c_{23}^{qR} e^{i\phi_{23}^{qL}} \\ c_{13}^{qL} s_{13}^{qR} c_{23}^{qL} e^{-i\phi_{13}^{qR}} & c_{13}^{qL} c_{13}^{qR} c_{23}^{qL} s_{23}^{qR} e^{-i\phi_{23}^{qR}} & c_{13}^{qL} c_{13}^{qR} c_{23}^{qL} c_{23}^{qR} \end{pmatrix}. \quad (34)$$

The left-right mixing angles θ_{LR}^q are given approximately as

$$\theta_{LR}^b \simeq \frac{m_b(A_{33}^{d,*} - \mu \tan \beta)}{m_{\tilde{b}_L}^2 - m_{\tilde{b}_R}^2}, \quad \theta_{LR}^t \simeq \frac{m_t(A_{33}^{u,*} - \mu \cot \beta)}{m_{\tilde{t}_L}^2 - m_{\tilde{t}_R}^2}. \quad (35)$$

Appendix B : Chargino and gluino interactions induced Z-penguin

The Z -penguin amplitude mediated by the chargino, $P_{\text{ZL}}^{sd}(\chi^\pm)$ in our basis [78] is given as follows:

$$P_{\text{ZL}}^{sd}(\chi^\pm) = \frac{g_2^2}{4m_W^2} \sum_{\alpha, \beta, I, J} (\Gamma_{\text{CL}}^{(d)\dagger})_{\alpha d}^I (\Gamma_{\text{CL}}^{(d)})_J^{\beta s} \left\{ \delta_I^J (U_+^\dagger)_\beta^1 (U_+)_1^\alpha [\log x_I^{\mu_0} + f_2(x_\alpha^I, x_\beta^I)] \right. \\ \left. - 2\delta_I^J (U_-^\dagger)_\beta^1 (U_-)_1^\alpha \sqrt{x_\alpha^I x_\beta^I} f_1(x_\alpha^I, x_\beta^I) - \delta_\beta^\alpha \left(\tilde{\Gamma}_L^{(u)} \right)_I^J f_2(x_I^\alpha, x_J^\alpha) \right\}, \quad (36)$$

where

$$(\Gamma_{\text{CL}}^{(d)})_I^{\alpha q} \equiv (\Gamma_L^{(u)} V_{\text{CKM}})_I^q (U_+)_1^\alpha + \frac{1}{g_2} (\Gamma_R^{(u)} \hat{f}_U V_{\text{CKM}})_I^q (U_+)_2^\alpha, \quad (37)$$

and

$$\left(\tilde{\Gamma}_L^{(u)} \right)_I^J \equiv \left(\Gamma_L^{(u)} \Gamma_L^{(u)\dagger} \right)_I^J, \quad (38)$$

with $q = s, d$, $I = 1 - 6$ for up-squarks, and $\alpha = 1, 2$ for charginos. Here, $(U_\pm)_i^\alpha$ denote the mixing parameters between the wino and the higgsino.

The right-handed Z penguin one, $P_{\text{ZR}}^{sd}(\chi^\pm)$ is also given simply by replacements between L and R , etc. [78].

The Z -penguin amplitude mediated the gluino, $P_{\text{ZL}}^{sd}(\tilde{g})$ [78] is written as follows:

$$P_{\text{ZL}}^{sd}(\tilde{g}) = -\frac{2}{3} \frac{g_3^2}{m_W^2} \sum_{I, J} (\Gamma_{\text{GL}}^{(d)\dagger})_d^I (\tilde{\Gamma}^{(d)})_I^J (\Gamma_{\text{GL}}^{(d)})_J^s f_2(x_I^{\tilde{g}}, x_J^{\tilde{g}}), \quad (39)$$

where

$$\left(\tilde{\Gamma}_R^{(d)} \right)_I^J \equiv \left(\Gamma_R^{(d)} \Gamma_R^{(d)\dagger} \right)_I^J. \quad (40)$$

The right-handed Z penguin $P_{\text{ZR}}^{sd}(\tilde{g})$ is also given simply by replacements between L and R .

Appendix C : Basic formulae

C1 : $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ and $K_L \rightarrow \pi^0 \nu \bar{\nu}$

The effective Hamiltonian for $K \rightarrow \pi \nu \bar{\nu}$ in the SM is given as [3]:

$$\mathcal{H}_{\text{eff}}^{\text{SM}} = \frac{G_F}{\sqrt{2}} \frac{2\alpha}{\pi \sin^2 \theta_W} \sum_{i=e, \mu, \tau} [V_{cs}^* V_{cd} X_c + V_{ts}^* V_{td} X_t] (\bar{s}_L \gamma^\mu d_L) (\bar{\nu}_L^i \gamma_\mu \nu_L^i) + \text{H.c.}, \quad (41)$$

which is induced by the box and the Z-penguin mediated the W boson. The loop function X_c denotes the charm-quark contribution of the Z-penguin, and X_t is the sum of the top-quark exchanges of the box diagram and the Z-penguin in Eq.(41).

Let us define the function F as follows:

$$F = V_{cs}^* V_{cd} X_c + V_{ts}^* V_{td} X_t. \quad (42)$$

The branching ratio of $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ is given in terms of F . Taking the ratio of it to the branching ratio of $K^+ \rightarrow \pi^0 e^+ \nu$, which is the tree level transition, we obtain a simple form:

$$\frac{\text{BR}(K^+ \rightarrow \pi^+ \nu \bar{\nu})}{\text{BR}(K^+ \rightarrow \pi^0 e^+ \nu)} = \frac{2}{|V_{us}|^2} \left(\frac{\alpha}{2\pi \sin^2 \theta_W} \right)^2 \sum_{i=e,\mu,\tau} |F|^2. \quad (43)$$

Here the hadronic matrix element has been removed by using the fact that the hadronic matrix element of $K^+ \rightarrow \pi^0 e^+ \nu$, which is well measured as $\text{BR}(K^+ \rightarrow \pi^0 e^+ \nu)_{\text{exp}} = (5.07 \pm 0.04) \times 10^{-2}$ [63], is related to the one of $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ with the isospin symmetry:

$$\langle \pi^0 | (\bar{d}_L \gamma^\mu s_L) | \bar{K}^0 \rangle = \langle \pi^0 | (\bar{s}_L \gamma^\mu u_L) | K^+ \rangle, \quad (44)$$

$$\langle \pi^+ | (\bar{s}_L \gamma^\mu d_L) | K^+ \rangle = \sqrt{2} \langle \pi^0 | (\bar{s}_L \gamma^\mu u_L) | K^+ \rangle. \quad (45)$$

Finally, the branching ratio for $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ is expressed as follows:

$$\text{BR}(K^+ \rightarrow \pi^+ \nu \bar{\nu}) = 3\kappa |F|^2, \quad \kappa = \frac{2}{|V_{us}|^2} r_{K^+} \left(\frac{\alpha}{2\pi \sin^2 \theta_W} \right)^2 \text{BR}(K^+ \rightarrow \pi^0 e^+ \nu), \quad (46)$$

where r_{K^+} is the isospin breaking correction between $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ and $K^+ \rightarrow \pi^0 e^+ \nu$ [46, 47], and the factor 3 comes from the sum of three neutrino flavors. It is noticed that the branching ratio for $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ depends on both the real and imaginary parts of F .

For the $K_L \rightarrow \pi^0 \nu \bar{\nu}$ decay, the $K^0 - \bar{K}^0$ mixing should be taken account, and one obtains

$$\begin{aligned} A(K_L \rightarrow \pi^0 \nu \bar{\nu}) &= \frac{G_F}{\sqrt{2}} \frac{2\alpha}{\pi \sin^2 \theta_W} (\bar{\nu}_L^i \gamma_\mu \nu_L^i) \langle \pi^0 | [F(\bar{s}_L \gamma_\mu d_L) + F^*(\bar{d}_L \gamma_\mu s_L)] | K_L \rangle \\ &= \frac{G_F}{\sqrt{2}} \frac{2\alpha}{\pi \sin^2 \theta_W} (\bar{\nu}_L^i \gamma_\mu \nu_L^i) \frac{1}{\sqrt{2}} [F(1 + \bar{\epsilon}) \langle \pi^0 | (\bar{s}_L \gamma_\mu d_L) | K^0 \rangle + F^*(1 - \bar{\epsilon}) \langle \pi^0 | (\bar{d}_L \gamma_\mu s_L) | \bar{K}^0 \rangle] \\ &\simeq \frac{G_F}{\sqrt{2}} \frac{2\alpha}{\pi \sin^2 \theta_W} (\bar{\nu}_L^i \gamma_\mu \nu_L^i) \frac{1}{\sqrt{2}} 2i \text{Im} F \langle \pi^0 | (\bar{s}_L \gamma_\mu d_L) | K^0 \rangle, \end{aligned} \quad (47)$$

where we use

$$|K_L\rangle = \frac{1}{\sqrt{2}} [(1 + \bar{\epsilon})|K^0\rangle + (1 - \bar{\epsilon})|\bar{K}^0\rangle], \quad (48)$$

with

$$\text{CP}|K^0\rangle = -|\bar{K}^0\rangle, \quad \langle \pi^0 | (\bar{d}_L \gamma_\mu s_L) | \bar{K}^0 \rangle = -\langle \pi^0 | (\bar{s}_L \gamma_\mu d_L) | K^0 \rangle. \quad (49)$$

We neglect the CP violation in $K^0 - \bar{K}^0$ mixing, $\bar{\epsilon}$, due to its smallness $|\bar{\epsilon}| \sim 10^{-3}$. Taking the ratio between the branching ratios of $K^+ \rightarrow \pi^0 e^+ \nu$ and $K_L \rightarrow \pi^0 \nu \bar{\nu}$, we have the simple form:

$$\frac{\text{BR}(K_L \rightarrow \pi^0 \nu \bar{\nu})}{\text{BR}(K^+ \rightarrow \pi^0 e^+ \nu)} = \frac{2}{|V_{us}|^2} \left(\frac{\alpha}{2\pi \sin^2 \theta_W} \right)^2 \frac{\tau(K_L)}{\tau(K^+)} \sum_{i=e,\mu,\tau} (\text{Im} F)^2. \quad (50)$$

Therefore, the branching ratio of $K_L \rightarrow \pi^0 \nu \bar{\nu}$ is given as follows:

$$\text{BR}(K_L \rightarrow \pi^0 \nu \bar{\nu}) = 3\kappa \cdot \frac{r_{K_L}}{r_{K^+}} \frac{\tau(K_L)}{\tau(K^+)} (\text{Im}F)^2, \quad (51)$$

where r_{K_L} denotes the isospin breaking effect [46, 47]. It is remarked that the branching ratio of $K_L \rightarrow \pi^0 \nu \bar{\nu}$ depends on the imaginary part of F .

The effective Hamiltonian in Eq.(41) is modified due to new box diagrams and penguin diagrams induced by SUSY particles. Then, the effective Lagrangian is given as

$$\mathcal{L}_{\text{eff}} = \sum_{i,j=e,\mu,\tau} [C_{\text{VLL}}^{ij} (\bar{s}_L \gamma^\mu d_L) + C_{\text{VRL}}^{ij} (\bar{s}_R \gamma^\mu d_R)] (\bar{\nu}_L^i \gamma_\mu \nu_L^j) + \text{H.c.}, \quad (52)$$

where i and j are the indices of the flavor of the neutrino final state. Here, $C_{\text{VLL,VRL}}^{ij}$ is the sum of the box contribution and the Z-penguin one:

$$C_{\text{VLL}}^{ij} = -B_{\text{VLL}}^{sdij} - \frac{\alpha_2}{4\pi} Q_{\text{ZL}}^{(\nu)} P_{\text{ZL}}^{sd} \delta^{ij}, \quad C_{\text{VRL}}^{ij} = -B_{\text{VRL}}^{sdij} - \frac{\alpha_2}{4\pi} Q_{\text{ZL}}^{(\nu)} P_{\text{ZR}}^{sd} \delta^{ij}, \quad (53)$$

where the weak neutral-current coupling $Q_{\text{ZL}}^{(\nu)} = 1/2$, and $B_{\text{VL(R)L}}^{sdij}$ and $P_{\text{ZL(R)}}^{sd}$ denote the box contribution and the Z-penguin contribution, respectively. The V , L and R denote the vector coupling, the left-handed one and the right-handed one, respectively. In addition to the W boson contribution, there are the gluino \tilde{g} , the chargino χ^\pm and the neutralino χ^0 mediated ones.

The branching ratios of $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ and $K_L \rightarrow \pi^0 \nu \bar{\nu}$ are obtained by replacing internal effect F in Eqs. (46) and (51) to $C_{\text{VLL}}^{ij} + C_{\text{VRL}}^{ij}$ as follows:

$$\text{BR}(K^+ \rightarrow \pi^+ \nu \bar{\nu}) = \kappa \sum_{i=e,\mu,\tau} |C_{\text{VLL}}^{ij} + C_{\text{VRL}}^{ij}|^2, \quad (54)$$

$$\text{BR}(K_L \rightarrow \pi^0 \nu \bar{\nu}) = \kappa \cdot \frac{r_{K_L}}{r_{K^+}} \frac{\tau(K_L)}{\tau(K^+)} \sum_{i=e,\mu,\tau} |\text{Im}(C_{\text{VLL}}^{ij} + C_{\text{VRL}}^{ij})|^2. \quad (55)$$

C2 : $B_s \rightarrow \mu^+ \mu^-$, $B^0 \rightarrow \mu^+ \mu^-$ and $K_L \rightarrow \mu^+ \mu^-$

The Z-penguin process appears in $B_s \rightarrow \mu^+ \mu^-$ and $B^0 \rightarrow \mu^+ \mu^-$ decays. We show the branching ratio for $B_s \rightarrow \mu^+ \mu^-$, which includes the Z-penguin amplitude [78]:

$$\text{BR}(B_s \rightarrow \mu^+ \mu^-) = \tau_{B_s} \frac{m_{B_s}^3 f_{B_s}^2}{16\pi} \left(\frac{\alpha}{4\pi} \right)^2 \frac{m_\mu^2}{m_{B_s}^2} \sqrt{1 - \frac{4m_\mu^2}{m_{B_s}^2}} \left| C_{\text{VRA}}^{(\mu)} - C_{\text{VLA}}^{(\mu)} \right|^2, \quad (56)$$

where

$$\frac{\alpha}{4\pi} C_{\text{VLA}}^{(\mu)} = -B_{\text{VLL}}^{(bs\mu\mu)}(SM) - \frac{\alpha_2}{4\pi} \frac{1}{4} P_{\text{ZL}}^{bs}, \quad \frac{\alpha}{4\pi} C_{\text{VRA}}^{(\mu)} = -\frac{\alpha_2}{4\pi} \frac{1}{4} P_{\text{ZR}}^{bs}. \quad (57)$$

We include the box diagram only for the SM, which is

$$B_{\text{VLL}}^{(bs\mu\mu)}(SM) = -\frac{\alpha_2}{4\pi} \frac{g_2^2}{2m_W^2} V_{tb} V_{ts}^* B_0(x_t). \quad (58)$$

On the other hand, the SM component of the Z-penguin amplitude is

$$P_{ZL}^{bs} = \frac{g_2^2}{2m_W^2} V_{tb} V_{ts}^* \times 4C_0(x_t), \quad (59)$$

where $B_0(x_t)$ and $C_0(x_t)$ are well known loop-functions depending on $x_t = m_t^2/m_W^2$. We have neglected other amplitudes such as the Higgs mediated scalar amplitude since we focus on NP in the Z-penguin process.

The branching ratio of $B^0 \rightarrow \mu^+ \mu^-$ is given in the similar expression. For the $K_L \rightarrow \mu^+ \mu^-$ decay, its branching ratio is given as follows [79] :

$$\text{BR}(K_L \rightarrow \mu^+ \mu^-)_{\text{SD}} = \kappa_\mu \left[\frac{\text{Re}\lambda_t}{\lambda^5} Y(x_t) + \frac{\text{Re}\lambda_c}{\lambda} P_c \right]^2, \quad (60)$$

$$\kappa_\mu = (2.009 \pm 0.017) \times 10^{-9} \left(\frac{\lambda}{0.225} \right)^8, \quad (61)$$

where λ is the Wolfenstein parameter, $\lambda_i = V_{is}^* V_{id}$ and the charm-quark contribution P_c is calculated in NNLO ; $P_c = 0.115 \pm 0.018$, and Y is the same as in eq.(7). We use its SM value as $Y(x_t) = 0.950 \pm 0.049$, ($x_t \equiv m_t^2/M_W^2$).

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